static characteristic impedances of the coupled microstrips [4]. The dielectric substrate thickness is H.

The previous considerations were used to obtain circuit models of the microstrip structures whose dispersion was measured and reported by Gould and Talboys [5]. The results comparing the experimental data of [5] and the dispersion calculated from our circuit model are shown in Fig. 2. For each set of microstrip dimensions labeled by a number, letters a and b refer to the dispersive odd and even TEM[1] modes, respectively, for that geometry as calculated from (3) using the plus sign, and with $p = j\omega$. The geometric and static data for the specific microstrips considered, taken from [4] and [5], are presented in Table I. The cutoff modes which are paired with the dispersive TEM modes are not shown in Fig. 2 since they are strongly attenuated over the frequency band shown; their cutoff frequencies are easily calculated from (3) as

$$f_{ci} = [K_{ci}v_0/2\pi(1 - k_i^2)^{1/2}\bar{\epsilon}_i^{1/2}]. \qquad (8)$$

Since the parameters of the approximation are based on the fundamental mode, this equation should probably be used with some caution. It has not been experimentally verified.

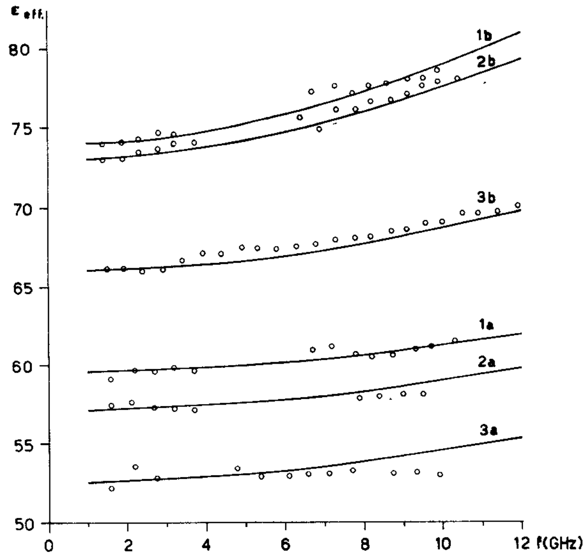

Fig. 2. Measured and calculated effective dielectric constant of TEM dispersive odd and even modes of parallel-coupled microstrips (see Table I for physical dimensions).

TABLE I
PARAMETERS OF PARALLEL-COUPLED MICROSTRIPS (AFTER GETSINGER [4])

Line	Mode	Static Impedance[a] Ω	Eff. Diel. Const.[b] at d.c.	W/H	S/H	H mm.
1a	odd	46.8	5.95			
1b	even	59.4	7.40	0.86	1.12	0.630
2a	odd	44.0	5.70			
2b	even	64.6	7.30	0.80	0.69	0.630
3a	odd	46.6	5.25			
3b	even	110.9	6.60	0.30	0.19	0.630

Notes: Column a calculated by the MSTRIP program using $\epsilon_s = 10.0$. Column b determined by extrapolation of experimental curves to zero frequency.

[1] These are not true TEM modes since they have a longitudinal H component, but they propagate to dc and for convenience are termed "dispersive TEM."

TABLE II
TE CUTOFF WAVENUMBERS OF PARALLEL-COUPLED MICROSTRIPS (ODD, TE_{01}; EVEN, TE_{10})

Line	Mode	$K_c = \pi/x$ (meters)$^{-1}$	x (mm.)	x/H
1a	odd	2066.6	1.52	2.41
1b	even	765.7	4.10	6.50
2a	odd	2061.6	1.52	2.41
2b	even	834.9	3.76	5.96
3a	odd	2233.1	1.41	2.24
3b	even	1391.9	2.26	3.60

The cutoff wavenumbers of the TE lines used in the model of the microstrips of Fig. 2 are reported in Table II. These numbers, K_{ce} and K_{co}, are parameters of the approximation and result from the assumption of using TE_{10} and TE_{01} as higher coupling modes. Thus they define the width and height of a hypothetical enclosing shield and, in effect, yield a plausible estimate of *equivalent shield* dimensions associated with the TE_{10}, TE_{01} cutoff wavenumbers. Thus from Tables I and II the equivalent height parameter (x/H, odd mode) is about 2.3 H for all geometries. The width parameter (x/H, even mode) varies from 6.5 H to 3.6 H (1.8/1), but note that S/H (S is conductor strip separation) varies over a 5.9/1 range. The substrate thickness H is 0.630 mm for all geometries.

In any case, just as in the dielectric loaded round guide [2], dielectric loaded rectangular guide [6], and single-strip microstrip [3], the coupled-line equivalent circuit gives a simple physical model with excellent experimental agreement for the dispersion properties of a parallel-coupled microstrip. We believe this further demonstrates that the coupled-line representation has broad applicability to a wide variety of longitudinally uniform, transversely inhomogeneous propagating structures.

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The Phase Shift Through Symmetrical 3-Port Circulators

G. RIBLET, MEMBER, IEEE

***Abstract*—Simple approximate formulas are derived for the phase shift through matched circulators—with and without transformer coupling—using expressions for the eigenadmittances Y_0, Y_{-1}, and Y_1 which have recently been proposed. These formulas allow one to predict the phase shift from measurements of the VSWR in one case and from a knowledge of the transformer admittance Y**

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The author is with the Microwave Development Laboratories Inc., Natick, Mass. 01760.

in the second. They have been confirmed experimentally for strip-line circulators and indicate that in this respect circulators behave like electrically long transmission devices.

I. INTRODUCTION

Circulators are frequently subject to a requirement on the transmission phase shift versus frequency in addition to requirements on VSWR, isolation, and insertion loss. It is well known from Bosma [1], for instance, that this phase shift is 180° at the center frequency of a lightly coupled circulator. This is insufficient information for most applications, however, as usually the variation of phase shift with frequency must be known over some frequency band. Little has appeared on this subject in the literature, however. One reason for this could be that the 2-port shunt resonator model which is commonly used to understand the VSWR [2], [3] of circulators is of no value when it comes to predicting this phase shift. In order to do this, one must have a model which treats the circulator as a 3-port device. Recently, Helszajn [2] has proposed such a model based on the eigenadmittances Y_0, Y_{-1}, and Y_1 [4], [5]. He assumes the following frequency dependences for the eigenadmittances Y_0, Y_{-1}, and Y_1 of a symmetrical 3-port circulator without external tuning elements

$$Y_0 = +\infty j$$

$$Y_{-1} = -j(b \cot \theta - g/\sqrt{3})$$

$$Y_1 = -j(b \cot \theta + g/\sqrt{3}). \qquad (1)$$

The quantity b is a normalized characteristic admittance, $\theta = (\pi/2)[1 + (\omega - \omega_0/\omega_0)]$ with ω_0 being the center frequency of the circulator, and ω, the operating frequency, and g is a monotonically increasing function of magnetic field. Equation (1) provides a phenomenological model for a circulator in terms of two quantities b and ω_0 which must be determined experimentally. The usual shunt resonator model also has two such quantities which likewise must be experimentally determined.

The important advantage of this 3-port model is that it allows all experimentally measurable properties of a circulator, including the transmission phase shift, to be calculated once b and ω_0 are known. In particular it allows the S-matrix parameters S_{11}, S_{12}, and S_{13} to be determined. These parameters depend on Y_0, Y_{-1}, and Y_1 through the following series of equations which are well known from the literature [6], [7]:

$$S_{11} = \frac{S_0 + S_{-1} + S_1}{3}$$

$$S_{12} = \frac{S_0 + S_{-1}\exp[-j(2\pi/3)] + S_1 \exp[j(2\pi/3)]}{3}$$

$$S_{13} = \frac{S_0 + S_{-1}\exp[j(2\pi/3)] + S_1 \exp[-j(2\pi/3)]}{3} \qquad (2)$$

and

$$S_0 = \frac{1 - Y_0}{1 + Y_0}$$

$$S_{-1} = \frac{1 - Y_{-1}}{1 + Y_{-1}}$$

$$S_1 = \frac{1 - Y_1}{1 + Y_1} \qquad (3)$$

with S_0, S_{-1}, and S_1 being the eigenvalues of the scattering matrix.

II. THE PHASE SHIFT THROUGH A SIMPLE MATCHED CIRCULATOR

In order to determine the phase shift ϕ, one must first determine S_{12}. The phase shift is then given by

$$\phi = \tan^{-1}\left\{\frac{\text{Im } S_{12}}{\text{Re } S_{12}}\right\}.$$

The eigenvalues S_0, S_{-1}, and S_1 will be of unit amplitude if the circulator is lossless so that the expression for S_{12} in (2) can be rewritten as

$$S_{12} = \tfrac{1}{3}\{\exp(j\psi_0) + \exp(j\psi_{-1})\exp[-j(2\pi/3)] + \exp(j\psi_1)\exp[+j(2\pi/3)]\}$$

where expressions for the phase angles ψ_0, ψ_{-1}, and ψ_1 follow from (3) and are

$$\psi_0 = -2\tan^{-1}(-jY_0), \qquad \psi_{-1} = -2\tan^{-1}(-jY_{-1}),$$
$$\psi_1 = -2\tan^{-1}(-jY_1).$$

Suppose we now assume that the circulator is well matched with the aim of finding simple approximate formulas for the phase shift. In this case S_{11} is small and the eigenvalues S_0, S_{-1}, and S_1 will be separated by close to 120° on the unit circle. One can write

$$\psi_{-1} = \psi_0 + 2\pi/3 + \delta_{-1} \qquad \psi_{+1} = \psi_0 - 2\pi/3 + \delta_1$$

with δ_{-1} and δ_1 being small angles. The preceding expression for S_{12} becomes

$$S_{12} = \frac{\exp(i\psi_0) + \exp[i(\psi_0 + \delta_{-1})] + \exp[i(\psi_0 + \delta_1)]}{3}$$

$$= \frac{\exp(i\psi_0)}{3}\{1 + \exp(i\delta_{-1}) + \exp(i\delta_1)\}$$

$$\simeq \frac{\exp(i\psi_0)}{3}\{3 + i(\delta_{-1} + \delta_1)\} \simeq \exp i\left(\psi_0 + \frac{\delta_{-1} + \delta_1}{3}\right).$$

Consequently, the phase shift ϕ is given approximately by

$$\phi \simeq \psi_0 + \frac{\delta_{-1} + \delta_1}{3} \qquad (4)$$

assuming again that δ_{-1} and δ_1 are small as they will be for a well-matched circulator. Equation (4) will be used to find approximate formulas for the phase shift through simple and transformer coupled circulators.

In the first case, expressions for δ_{-1} and δ_1 can be found using the equations

$$-jY_{-1} = \tan\{-\psi_0/2 - \pi/3 - \delta_{-1}/2\}$$

$$-jY_1 = \tan\{-\psi_0/2 + \pi/3 - \delta_1/2\}$$

with $\psi_0 = \pi$ since $Y_0 = j\infty$. To determine δ_{-1} and δ_1, it is necessary to substitute (1) for Y_{-1} and Y_1 with g set to 1 so that the circulator is matched at midband, to expand the tan function using trigonometric formulas, and then to solve for $\tan(\delta_{-1}/2)$ and $\tan(\delta_1/2)$ which can be approximated by $\delta_{-1}/2$ and $\delta_1/2$, respectively, if these quantities are small. The result of these calculations is

$$\frac{\delta_{-1} + \delta_1}{3} \simeq b \cot \theta.$$

From (4) it follows that

$$\phi \simeq \pi + b \cot \theta. \qquad (5)$$

Now b can be determined from the frequency dependence of the input VSWR either by curve fitting or by using the formula $b = (4/\pi\sqrt{3})[\omega_0/(\omega_{+1} - \omega_{-1})]$ where ω_{+1} and ω_{-1} are the upper and lower frequencies at which the VSWR = 2, [2], [5]. Once this quantity has been found, the phase shift can be determined from (5).

This procedure was carried through with a 3.7–4.2-GHz stripline circulator. Measurements of return loss and phase were made with a high-pass (HP) network analyzer. A phase reference was established by placing a short circuit at the same location as the ferrite boundary. The phase shift is taken from the ferrite boundary in this case and the outer transformer junction in the case of the transformer coupled circulator. In Fig. 1,the measured return loss is given as a function of frequency. The dashed curves were calculated from (1)–(3) using values for b of 5.5 and 6.5. There is a good fit to the experimental curve with b = 6.5 while b = 5.5 is clearly too small. In Fig. 2 the measured phase shift is given as a function of frequency. The dashed curve follows from (5) with b = 6.5. The agreement is quite good. It should be pointed out that formula (5) agrees with the exact phase shift which can be calculated from (1)–(3) within 2 percent over the frequency range 3.5–4.3 GHz.

III. THE PHASE SHIFT THROUGH A TRANSFORMER COUPLED CIRCULATOR

A simple approximate formula for the phase shift can also be derived for the case of a circulator which is matched with identical transformers connected at each port. In this case the result follows almost directly from (4). If the circulator is perfectly matched at two frequencies in the passband as a result of the matching transformers, then δ_{-1} and δ_1 will be zero at these two frequencies. It can also be shown that for this model $\delta_{-1} + \delta_1 = 0$ at the center frequency. It is reasonable to take $\delta_{-1} + \delta_1 \simeq 0$ in the passband. It follows from (4) that

$$\begin{aligned}\phi &\simeq \psi_0 \\ &\simeq -2 \tan^{-1} (Y_0'). \end{aligned} \tag{6}$$

Y_0' is the input admittance for the symmetrical eigenexcitation at the junction to the transformers. It is easy to show that

$$Y_0' = \frac{C + DY_0}{A - BY_0}$$

with A, B, C, D being the elements of the transfer matrix of the transformer which has been connected at each port. Y_0 is the input admittance at the ferrite junction which has been assumed to be approximately given by $j\infty$. Consequently,

$$Y_0' = -D/B = -Y \cot \theta$$

if the transformers are a quarter wavelength long at midband with characteristic admittance Y. Finally, substituting this expression into (6), one has

$$\phi \simeq -2 \tan^{-1} (-Y \cot \theta). \tag{7}$$

Surprisingly, the phase shift depends on the characteristic admittance of the transformer alone.

Phase measurements were made on the same stripline circulator used in the previous measurements except now with transformer dielectrics giving a characteristic admittance Y = 1.82 in each of the three arms. The VSWR was less than 1.4 from 3.0 to 5.0 GHz. In Fig. 3 the experimental values of the transmission phase are given over this frequency range along with values calculated from (7) with Y = 1.82. The phase contribution due to connectors and connecting strips has been subtracted out. The agreement is reasonably good although it could be improved with a somewhat larger value for Y.

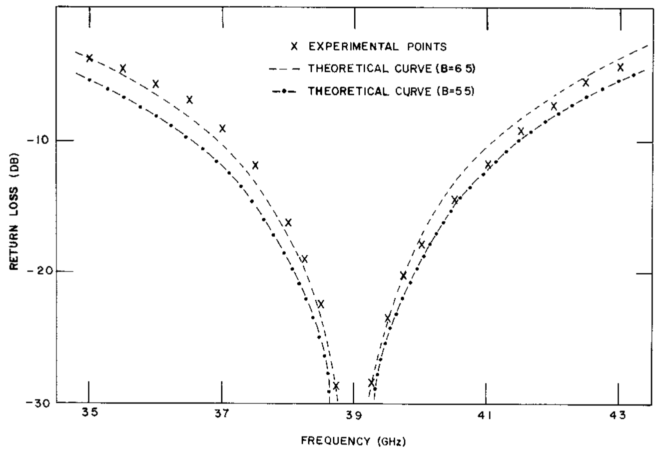

Fig. 1. The return loss of a simple stripline circulator as a function of frequency. The crosses are experimental points while the dashed curves were calculated with values for b = 5.5 and 6.5.

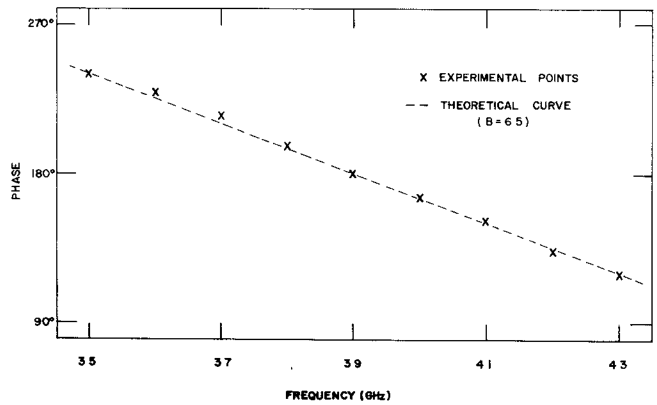

Fig. 2. The transmission phase shift of a simple stripline circulator as a function of frequency. The crosses represent experimental points while the dashed curve was calculated from (5) with b = 6.5.

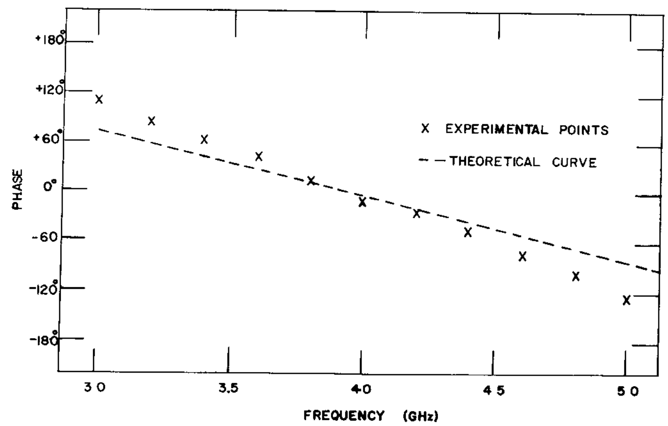

Fig. 3. The transmission phase shift of a transformer matched circulator as a function of frequency. The crosses represent experimental points while the dashed curve was calculated from (7) taking Y = 1.82.

IV. CONCLUSIONS

It is apparent from Figs. 2 and 3 that the transmission phase for both kinds of circulators is a nearly linear decreasing function of frequency. Such is also the case for a length of transmission line L. Let us write the phase shift as a function of wavelength $\phi(\lambda)$ as

$$\phi(\lambda) = -(2\pi/\lambda)L_{\text{eff}} + \phi_0$$

with ϕ_0 being a constant and L_{eff} an effective free-space line length for the circulator. Then one can show from (5) that for a simple circulator

$$L_{\text{eff}} = bc/4\omega_0 \tag{8}$$

with c being the speed of light. In other words the effective length increases as b increases or as the bandwidth of the device decreases. Similarly, for a transformer matched circulator it follows from (7) that

$$L_{eff} = Yc/2\omega_0. \quad (9)$$

For the circulators tested, (8) yields an effective length L_{eff} = 12.7 cm in the first case, while (9) yields an effective length L_{eff} = 6.8 cm in the second. Surprisingly, the effective length of the transformer matched circulator is smaller although it includes two additional quarter-wavelength sections of transmission line. The equivalent free-space diameter of the garnet disk was 5.2 cm or much less than the effective length of 12.7 cm for the simple circulator. Clearly, circulators behave in this respect like electrically long transmission devices.

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Power Generation and Efficiency in GaAs Traveling-Wave Amplifiers

F. GIANNINI AND A. SALSANO, MEMBER, IEEE

***Abstract*—The effect of the dielectric loading in a bidimensional GaAs traveling-wave amplifier (TWA) is investigated, with respect to the EM power generated by the structure and the efficiency of the dc to RF conversion. The validity of some usual approximations and assumptions is studied and a parameter, i.e., the power gain × efficiency product, is proposed as a useful tool for comparing the possible performances of TWA's.**

In this short paper the authors study the traveling-wave amplifier (TWA) structure proposed in Fig. 1 and seek an expression of the EM power generated by the electronic beam, which starts from the definition of an equivalent negative conductivity of the medium, which takes into account the effect of the RF charge ρ.

Following such an approach and using the solutions of dispersion relationship associated with the structure, the theoretical efficiency for different lateral loadings is obtained and the power gain × efficiency is indicated as a meaningful parameter of the TWA.

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The authors are with the Institute of Electronics, Faculty of Engineering, University of Rome, Rome, Italy.

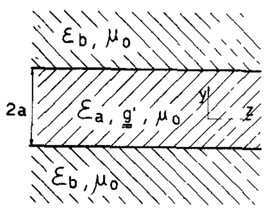

Fig. 1. Geometric behavior of the structure. The values of parameters used in the calculation of Re $[g_z']$ are: ϵ_a = 12, a = 1 μm, $n_0 = 10^{15}$ cm^{-3}, $v_0 = 1.5 \times 10^7$ cm/s, μ_y = 7200 $cm^2/(V\cdot s)$, μ_z = − 2400 $cm^2/(V\cdot s)$.

The propagation of EM waves in negative differential mobility (NDM) media has been studied by many authors [1]–[4] both in monodimensional and bidimensional approximation.

These papers deal prevalently with the modal solution of the structure which can sustain an infinite number of modes having different complex propagation constants, some of which correspond to growing waves. Following such an approach, the stability conditions were also ascertained which allow the use of the active structure as a TWA.

On the other hand, less attention has been paid to the energy as an approach which takes into account the power balance between the electronic stream and the EM field [5].

This is, moreover, a very important aspect of the matter because, starting from the premise that the real power "delivered" to the beam equals the real part of the Poynting vector flux entering a close surface delimiting a volume τ, the existence or nonexistence of amplifying waves is connected to the "sign" of the power "delivered" to the electronic beam. So, a formulation in terms of real power associated with the beam, except the kind of instability the beam can support, i.e., an absolute or convective instability, gives useful information on the properties of the structure when it is used as an amplifier.

Nevertheless, once it is found by the usual methods (Briggs [7], for instance) that instabilities are of the convective kind, the amount of generated real power for a fixed dc power dissipated by the device gives directly the efficiency of dc to RF power conversion.

The simple and well-known monodimensional model confirms that this point of view is essential for the evaluation of the amplifier characteristics and is a necessary complement of the solution of the problem in terms of phase and amplification constants associated with the space-charge wave.

In fact, an infinite medium with uniform characteristics is considered in the usual hypothesis of the small-signal traveling-wave analysis that follows.

1) Carrier mean free path is much shorter than wavelength.

2) Carrier lattice collision frequency is very large compared with the operating frequency.

3) Carrier drift velocity is parallel to the direction of wave propagation.

A compatible solution of the problem is [1]

$$\beta = \beta_e - j\beta_{cz}$$

where

$$\beta_e = \omega/v_0$$

$$\beta_{cz} = en_0\mu_z/\epsilon_0\epsilon_a v_0$$

with n_0 doping density, μ_z mobility, ϵ_a permittivity of the medium.

If μ_z is negative, as it can be in the case of GaAs devices biased above threshold, the corresponding wave grows as it travels at the "greatest growth rate" obtainable [2] in a monodimensional NDM structure, but no real RF power is generated since RF current density and electric field are 90° out of phase in the time domain, in accord-